Hayfield Secondary
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Hayfield Secondary AP Summer Assignment Cover Sheet

Course	
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Assignment Title	AB/BC Calculus Summer Assignment BC has ONE section that is BC only. Clearly labeled on the assignment. Otherwise, the assignments are the same. Calculus is Calculus! AND: This assignment is based on Algebra and Pre-Calculus, you do not need to learn or use calculus to complete the assignment!! Please do all work in a composition book.
Date Assigned	June 2019
Date Due	First Day of 2nd week of Class: 2019-2020 School year
Objective/Purpose of Assignment	**Review material from Previous Math classes that will be needed for success in AB/BC CALCULUS**
Description of how Assignment will be Assessed	**We will randomly choose 10 of the problems to grade for correctness (one question from each section). They will be graded as 1 point each. The rest of the grade is based on completeness.** **Graded down for being late. You will lose 10%/day up to 20% off.** **Graded down for not being in a composition book.**
Grade Value of Assignment	**30 points. This is similar to a large quiz. Class is graded on total points. Test are 100 points.**
Tools/Resources Needed to Complete Assignment	**Old Notes from previous classes (precalculus and Algebra 2 material ON LINE SITES . PatrickJMT is great!) http://patrickjmt.com/**
Estimated Time Needed to Complete	3-6 hours depending on how much you remember

Suggested supplies for class:
At least one composition book for homework/summer assignment: Do summer assignment in Composition Book and then use the same book for HOMEWORK ONLY.
A notebook for class notes (3 ring or composition book)
A 3 ring binder to keep all handouts/quizzes/test.
Dry erase markers for use in class

Hayfield SS: Summer assignment for AB/BC Calculus: 30 points
NOTES:

- **Section IV is for BC students only**. **ALL other sections are for both classes.**
- Section XI is for studying/memorizing only. There is no written work.
- Problems should ALL BE DONE in a **composition notebook** and the sections, letters should be CLEARLY labeled.
- You will be graded on your work, completeness and accuracy.
- All late work will be graded down 10% per school day (not class meeting days). You could lose up to 20%. This would give you 80% of the grade on the assignment. Assignments must be turned in before they are returned to the class.

I For each of the following functions

- ➢ Make a table of values for the function that clearly shows the main points.
- ➢ Sketch the graph for the function. When you sketch the graph be sure that all the points in your table are clearly shown on the graph. That is: if it is in the table, I want to see the POINT (a dot) on the graph

Please include **AT LEAST 6 points in your table**. These important points should include any key features such as:

- The roots (x-intercepts)
- Y-intercepts
- Asymptotes : vertical and horizontal
- Local maximum and minimum values.
- Period if applicable

Your table should be big enough to illustrate these features
Also label the following:

- Label by name any local maximum or minimum values eg. This is a **local max** and use an arrow to point to the dot on the graph.
- Sketch asymptotes with a "broken line".
- Show in the table the roots and y-intercept

Here are your functions:

a) $y = \dfrac{x+2}{x^2 - 6x + 5}$

b) $y = e^{3x} - 1$

c) $y = e^{-x^2}$

e) $y = 2\sin(\dfrac{x}{3} + \dfrac{\pi}{4})$

Be sure to do this problem in RADIANS: Leave points in terms or pi

II For each of the following, find the **area** described.

- Be sure to sketch the picture described. Draw the functions for the given domain
- Shade the region enclosed by the functions described.
- Show how you calculated the total area described.

a) The area **enclosed by** the following lines:
- $y = 2x$
- $y = 3$
- $y = -2x + 8$
- The horizontal x-axis ($y = 0$)

b) The area enclosed by the functions
- $y = \sqrt{9 - x^2}$
- The horizontal x-axis ($y = 0$)

c) Use 2 rectangles to estimate the area enclosed by the graphs of:
- $y = 2^x$
- $x = 0$
- $x = 3$
- The x-axis: $y = 0$

Give an explanation for your estimate and tell me if the estimate is TOO HIGH or TOO LOW. (Hint: draw rectangles inside or outside of your picture and use a calculator to help you find the areas of the rectangles)

III Find the "inverse function" for each of the following:
Show all work for FINDING the inverse function.
Graph the inverse function and the original function on the same axis (x-y plane)
You should also draw the line $y = x$ as a broken line. Recall that a function and its inverse are symmetrical about the line $y = x$.

a) $y = 2x^2 - 1; \quad x \geq 0$

b) $y = e^{2x}$

c) $y = \cos x; \quad 0 \leq x \leq \pi$

IV **<u>THIS SECTION IS BC ONLY: AB: continue on to section V</u>**

Summation: Find the following sums and answer the related questions:

a) $\sum_{n=1}^{32} \frac{1}{n} =$

What do you think will happen to the sum as n gets larger?
Hint: Consider the sum of the first 64 terms!

b) $\sum_{n=1}^{16} \frac{1}{n^2} =$

What do you think will happen to the sum as n gets larger?

c) $(1+\frac{1}{n})^n =$

What do you think will happen to the terms of this sequence as n gets larger?

Do this for the following values to try to get a "feel" for what is happening: Use these values of n: 1, 10, 100, 1000, 10 000, 100 000 This should give you an estimate for the number.

Do you recognize the number you see?

What number is this approaching as n approaches infinity?

NOTE ABOUT c) above:

Recall:

- As n gets very large, $\frac{1}{n} \to 0$
- As n gets very large, $(1.00001)^n \to \infty$ That means the expression get REALLY big for big values of "n"
- $(1)^n = 1$ for any value of n.
- Notice the above answer shows that some of the obvious rules may not apply when they are in conflict in the same problem.

V Find the following composition of functions:

Given the following:

$$f(x) = \sin x, \quad g(x) = 2x^2 - 3x + 1, \quad h(x) = \frac{5}{x+1}$$

Find:

a) $f(h(x)) =$

b) $g(h(x)) =$

c) $h(g(x)) =$

VI Write the following function as the composition of TWO functions: (decompose into two functions)

Ex: $h(x) = \frac{3}{x^2 + 5x}$ is the composition:

$h(x) = f(g(x))$ where: $f(x) = \frac{3}{x}$, and $g(x) = x^2 + 5x$

Let f(x) be the "big picture) and g(x) be the details within

a) $h(x) = (x^2 + 5x + 6)^4$

b) $h(x) = \sin(x^2 + 6x + 1)$

c) $h(x) = \frac{6}{(3x+5)^2}$

VII Add/Subtract or Multiply/Divide the following:

a) $\frac{3}{x+4}+\frac{5}{x-2}=$

b) $\frac{2}{(x-3)}+\frac{5}{x^2-6x+9}=$

c) $\frac{2}{5}+\frac{1}{2}=$

d) $\frac{\frac{4}{9}x^{\frac{3}{2}}}{\frac{3}{2}}=$

e) $x^{\frac{1}{2}}\cdot x^{\frac{2}{3}}=$

f) $\sqrt{x}\cdot(x^2+x^3\sqrt{x})$

g) $e^x e^2=$

h) Simplify as a single product or sum: $10^{k+3}=$

VIII Solve for y:

a) $\ln y = x+2$

b) $3xy+5y=2x+7$

c) $3x^2y-18=2xy+5x^2$

IX. **Graphs/Properties of Functions:** You may want to organize the following in a way you can save and use for a reference for the year. This section does not need to be IN THE COMPOSITION BOOK if you choose, but it can be. It must still be turned in.

Fill in the following table AND GRAPH each of the following.
Do Graphs on separate page!

	The Function	Domain	Range	Type of Symmetry: x-axis, y-axis, origin	Odd, Even, Neither	Intervals of Increasing/Decreasing
a)	$y = 1$					
b)	$y = x$					
c)	$y = x^2$					
d)	$y = x^3$					
e)	$y = x^4$					
f)	$y = \sqrt{x}$					
g)	$y = \sqrt[3]{x}$					
h)	$y = \frac{1}{x}$					
i)	$y = \frac{1}{x^2}$					
j)	$y = \|x\|$					
k)	$y = [\![x]\!]$					
l)	$y = \ln x$					
m)	$y = e^x$					
n)	$y = \sin x$					
o)	$y = \cos x$					

X Sketch a graph with the following properties:

a) Sketch a piece of a graph that is increasing and concave up.
*Make up a scenario/situation that may be reasonably represented by this graph! BE CREATIVE!

b) Sketch a piece of a graph that is increasing and concave down.
Make up a scenario/situation that may be reasonably represented by this graph!

c) Sketch a piece of a graph that is decreasing and concave up.
Make up a scenario/situation that may be reasonably represented by this graph

d) Sketch a piece of a graph that is decreasing and concave down.
Make up a scenario/situation that may be reasonably represented by this graph

e) For all values of the domain, x, the function $f(x) > 0$, $f(x)$ is increasing for all values of x and $f(x)$ is concave up for all values of x.

f) $f(0) = 2$, $f(x)$ is increasing for all x >0, $f(x)$ is concave down for all x >0.

g) $f(3) = 5$, $f(x)$ is increasing for all x <3, $f(x)$ is decreasing for all x >3. $f(x)$ is a continuous (no breaks in the graph).

h) $f(3) = 5$, $f(x)$ is increasing and concave up for all x <3, $f(x)$ is decreasing and concave down for all x >3. $f(x)$ is a continuous (no breaks in the graph).

i) The population is growing at a rate that is directly proportional to the current population. That is: as the population increases, so does the rate of growth for the population. THIS IS EXPONENTIAL GROWTH

XI

The following s**hould be familiar. <u>Know</u> the Pythagorean and Reciprocal Identities**

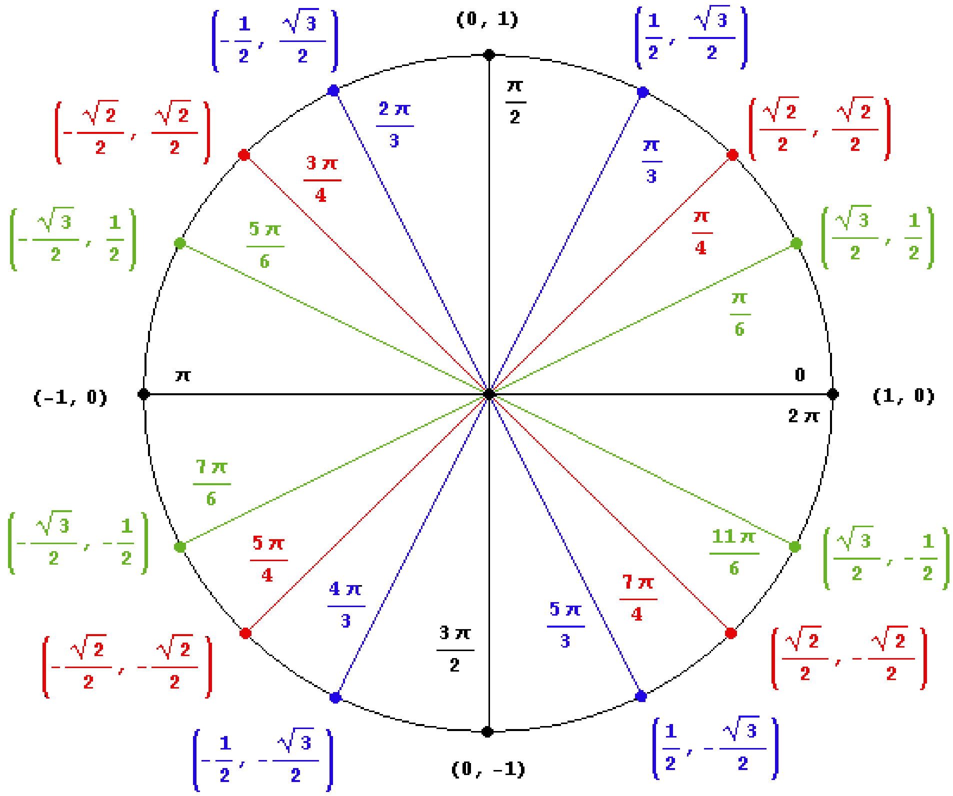
Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\sin x = \frac{1}{\csc x}$ $\quad \csc x = \frac{1}{\sin x}$ $\cos x = \frac{1}{\sec x}$ $\quad \sec x = \frac{1}{\cos x}$ $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Co-Function Identities	Odd/Even Identities
$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$	<u>Odd</u> $\sin(-\theta) = -\sin\theta$ $\csc(-\theta) = -\csc\theta$ $\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$ <u>Even</u> $\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$

Double Angle Identities	*Half Angle Identities*
$\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 2\cos^2 x - 1$ $\cos 2x = 1 - 2\sin^2 x$	$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\cos^2 x = \frac{1 + \cos 2x}{2}$

The Radian Measures and Coordinates **MUST** be **memorized**

Remember: $\sin\theta = \frac{y}{r} = y - coordinate$, $\cos\theta = \frac{x}{r} = x - coordinate$, and $\tan\theta = \frac{y}{x} = \frac{y - coordinate}{x - coordinate}$



KNOW AND UNDERSTAND ALL THE FOLLOWING FORMULAS and their uses.

Equation of a line:

$$y = mx + b$$
$$y - y_1 = m(x - x_1)$$
$$Ax + By = C$$

Slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel lines have the same slope
Perpendicular lines have slopes that are “negative reciprocals”

Area of a rectangle is length x width

BE PREPARED FOR A TEST on ALL the above the second week of class.

MATERIALS FOR CLASS.
You should have

- Composition book for homework
- Notebook for class notes as well as a ring binder or large trapper to keep test, quizzes, handouts, etc.
- Dry erase markers so that we can use individual white boards for class practice.
- A Graphing Calculator